Math 340 Spring 2014 HW6 due in discussion the week of March 3-7, 2014 Determinants

Remark. Answers (except for question 4) should be written in the following format: A) Result.

B) If possible the name of the method you used.

C) The computation.

1. Computing Determinants.

For the following matrices A, compute the determinant det(A).

(a)
$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$
.
(b) $A = \begin{pmatrix} -3 & 2 \\ 6 & 4 \end{pmatrix}$.
(c) $A = \begin{pmatrix} 1 & 1 & 4 \\ 2 & -3 & 5 \\ 4 & 2 & -2 \end{pmatrix}$.
(d) $A = \begin{pmatrix} 1 & 0 & -2 \\ 3 & 2 & 3 \\ 5 & 4 & 7 \end{pmatrix}$.
(e) $A = \begin{pmatrix} 1 & 1 & 0 & -2 & 3 \\ 2 & -1 & 3 & 1 & -2 \\ 4 & 2 & 7 & -2 & 1 \\ 2 & 4 & 1 & 2 & 0 \\ 0 & -2 & 4 & -2 & 3 \end{pmatrix}$.

2. Determinants and Invertibility.

For the following matrices A, use the determinant det(A) to decide whether or not there exists A^{-1} .

(a)
$$A = \begin{pmatrix} -2 & 4 \\ 3 & 3 \end{pmatrix}$$
.
(b) $A = \begin{pmatrix} 1 & 4 & -2 \\ 3 & 1 & 4 \\ 4 & 5 & 2 \end{pmatrix}$.
(c) $A = \begin{pmatrix} -2 & 3 & 1 \\ 2 & 8 & 2 \\ 0 & 11 & 5 \end{pmatrix}$.

3. Properties of the Determinant.

- (a) Show that for all matrices $A, B \in M_n$, $\det(AB) = \det(BA)$ (Hint: $\det(AB) = \det(A) \cdot \det(B)$).
- (b) Show that if $B = CAC^{-1}$, then det(A) = det(B).
- (c) Show that $det(\alpha A) = \alpha^n det(A)$ for every $\alpha \in \mathbb{R}$, and $A \in M_n$.
- (d) Give example of $A, B \in M_2$ such that $\det(A + B) \neq \det(A) + \det(B)$.
- 4. Answer the following:
 - (a) Write down the three properties that the function det : $M_n \to \mathbb{R}$, satisfies with respect to row operations.
 - (b) Describe the algorithm to compute the determinant of a matrix $A \in M_n$.

Remarks

- You are very much encouraged to work with other students. However, submit your work alone.
- The TA and the Lecturer will be happy to help you with the homework. Please visit the office hours.

Good Luck!