Math 340 - Solutions - HW #6

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1) a) $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \xrightarrow{1} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ Det(A) = 1

b) $\begin{pmatrix} -3 & 2 \\ 6 & 4 \end{pmatrix} \xrightarrow{1} \begin{pmatrix} -3 & 2 \\ 0 & 8 \end{pmatrix}$ Det(A) = -24

c) $\begin{pmatrix} 1 & 1 & 4 \\ 2 & -3 & 5 \\ 4 & 2 & -2 \end{pmatrix} \xrightarrow{1} \begin{pmatrix} 1 & 1 & 4 \\ 0 & -5 & -3 \\ 0 & -2 & -18 \end{pmatrix} \xrightarrow{2} \begin{pmatrix} 1 & 1 & 4 \\ 0 & -10 & -6 \\ 0 & -2 & -18 \end{pmatrix}$

$\xrightarrow{-5} \begin{pmatrix} 1 & 1 & 4 \\ 0 & -10 & -6 \\ 0 & 10 & 90 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 4 \\ 0 & -10 & -6 \\ 0 & 0 & 84 \end{pmatrix}$ $\text{Det}(A) = \dfrac{-840}{(-5)(2)} = 84$

d) $\begin{pmatrix} 1 & 0 & -2 \\ 3 & 2 & 3 \\ 5 & 4 & 7 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 9 \\ 0 & 4 & 17 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -2 \\ 0 & 2 & 9 \\ 0 & 0 & -1 \end{pmatrix}$

Det(A) = -2

e) Det(A) = -468

2) a) $\begin{pmatrix} -2 & 4 \\ 3 & 3 \end{pmatrix} \xrightarrow{2} \begin{pmatrix} -2 & 4 \\ 6 & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} -2 & 4 \\ 0 & 18 \end{pmatrix}$

$\text{Det}(A) = \dfrac{-2 \cdot 18}{2} = -18 \neq 0$ A is invertible.

b) $\begin{pmatrix} 1 & 4 & -2 \\ 3 & 1 & 4 \\ 4 & 5 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 4 & -2 \\ 0 & -11 & 10 \\ 0 & -11 & 10 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 4 & -2 \\ 0 & -11 & 10 \\ 0 & 0 & 0 \end{pmatrix}$

Det(A) = 0 A is not invertible.

c) $\begin{pmatrix} -2 & 3 & 1 \\ 2 & 8 & 2 \\ 0 & 11 & 5 \end{pmatrix} \longrightarrow \begin{pmatrix} -2 & 3 & 1 \\ 0 & 11 & 3 \\ 0 & 11 & 5 \end{pmatrix} \longrightarrow \begin{pmatrix} -2 & 3 & 1 \\ 0 & 11 & 3 \\ 0 & 0 & 2 \end{pmatrix}$

$\text{Det}(A) = -44 \neq 0$ A is invertible.

3) a) $\text{Det}(AB) = \text{Det}(A)\text{Det}(B) = \text{Det}(B)\text{Det}(A) = \text{Det}(BA)$

b) Let

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \qquad BA = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\text{Det}(AB) = \text{Det}(BA) = \underline{1}$$

c) $B = CAC^{-1}$. $\text{Det}(B) = \text{Det}(CAC^{-1}) = \text{Det}(C)\text{Det}(A)\text{Det}(C^{-1})$

$= \text{Det}(A)\,\text{Det}(C)\,\text{Det}(C^{-1}) = \text{Det}(A)\,\text{Det}(C \cdot C^{-1}) = \text{Det}(A)$