Math 340 Spring 2014
HW5, due in discussion Feb. 24-28 2014
Inverse Matrices and Transformations

Remark. Answers (except for 1) should be written in the following format:
A) Result.
B) If possible the name of the method you used.
C) The computation.

Definitions.
Let $f : X \to Y$ be a function.

- Define what it means for f to be <u>one-to-one</u>.
- Define what it means for f to be <u>onto</u>.
- Define what it means for f to be <u>invertible</u>.

1. **Invertible Matrices and Invertible Transformations**

   Let A be an $n \times n$ matrix.

   (a) Show that if A is invertible then the transformation (function) T_A is invertible.

   (b) Show that if T_A is invertible then A is invertible (Hint: For every $\mathbf{y}$ there is an $\mathbf{x}$ such that $T_A(\mathbf{x}) = \mathbf{y}$. In addition, you can use the fact that if there is $B \in M_n$ such that $AB = I_n$ then A is invertible).

   (c) Suppose there exists a vector $\mathbf{v} \in Ker(A) = \{\mathbf{x} \in \mathbb{R}^n |\ A\mathbf{x} = 0\}$ that is not the zero vector. Show that A is not invertible.

2. **The Inverse of a Product of Matrices**

   (a) Let
   $$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix},\ B = \begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix}.$$
      1. Compute A^{-1} and B^{-1}.
      2. Compute $C = B^{-1}A^{-1}$.
      3. Compute ABC.

   (b) Suppose A and B are invertible $n \times n$ matrices. Show that AB is invertible.

   (c) Suppose that AB is invertible. Show that A and B are invertible.

3. **Inverting Matrices**

   Use the methods discussed in class (row reduction) to determine if the matrices below are invertible. If so, give the inverse.

(a)

$$A = \begin{pmatrix} -2 & 4 \\ 3 & 5 \end{pmatrix}.$$

(b)

$$B = \begin{pmatrix} 1 & 1 & -1 \\ 4 & 6 & -2 \\ 1 & 3 & 1 \end{pmatrix}.$$

(c)

$$C = \begin{pmatrix} 3 & 5 & 0 \\ 1 & 4 & 2 \\ 4 & -2 & 2 \end{pmatrix}.$$

4. **Arithmetic Complexity of Inversion By Row Reduction**

   If we define one "operation" to be the addition, subtraction, multiplication, or division of two real numbers, for example $3 + 5$ or $17/3$, approximately how many operations, in terms of n, does our algorithm for inverting an $n \times n$ matrix require (Just give the degree of the polynomial, for instance if it takes $3n^5 + 2n^4 + 8$ operations, the answer is n^5). Justify your answer.

Remarks

- You are very much encouraged to work with other students. However, submit your work alone.
- The TA and the Lecturer will be happy to help you with the homework. Please visit the office hours.

Good Luck!