Math 340 Spring 2014
Homework #4, due in the week of Feb. 17-21
Matrices and Transformations

Remark. Answers (except for questions 3,4,5) should be written in the following format:
A) Result.
B) If possible the name of the method you used.
C) The computation.

1. **MATRICES AND TRANSFORMATIONS**

   (a) Let
   $$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}, \ e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$
   Compute $T_A(e_1)$, $T_A(e_2)$, and $T_A(e_3)$. Remember that $T_A(v) = Av$.

   (b) Let
   $$B = \begin{pmatrix} 3/2 & -1/2 & 1/2 \\ -1 & 2 & 1 \\ -1/2 & 1/2 & 5/2 \end{pmatrix}, \ f_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \ f_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \ f_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$
   Compute $T_B(f_1)$, $T_B(f_2)$, and $T_B(f_3)$.

   (c) Let
   $$v = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \in \mathbb{R}^3.$$
   Find $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$ so that $v = \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3$.

   (d) Let
   $$u = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$
   Compute $T_B(u)$ two ways. First compute it with matrix multiplication. Then compute it using the information from parts (b) and (c).

2. **Exponentiating Matrices**
   Let
   $$A = \begin{pmatrix} 5/2 & 3/2 \\ -1/2 & 1/2 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}.$$

   (a) Compute B^{-1}.

(b) Compute $D = BAB^{-1}$.

(c) Compute A^{13} (A multiplied by itself 13 times). Hint: $A^2 = B^{-1}D^2B$. Note: Expressions of the form 2^{13} may appear in your answer.

3. Let $A \in M_{m\times n}(\mathbb{R})$. Define the linear transformation $T_A : \mathbb{R}^n \to \mathbb{R}^m$, associated with A.

4. Let $f : X \to Y$, and $g : Y \to Z$, be two functions. Define their composition, $g \circ f : X \to Z$.

5. Define what it means for two matrices $A, B \in M_n(\mathbb{R})$ to be conjugate.

Remarks

- You are very much encouraged to work with other students. However, submit your work alone.
- The TA and the Lecturer will be happy to help you with the homework. Please visit the office hours.

Good Luck!