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Math 340 - Solutions - HW #3

1) a) $R_0 v = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad R_{(2\pi/3)} v = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} v = \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \end{pmatrix}$

$R_{(4\pi/3)} v = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} v = \begin{pmatrix} -1/2 \\ -\sqrt{3}/2 \end{pmatrix}$ $\quad R_{(2\pi)} v = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

b) $R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\quad S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad RS = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad SR = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

2) a) $A = \begin{pmatrix} 1 & 0 & -2 \\ 1 & 3 & 1 \\ 2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{pmatrix}$ If $A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \vec{b}$, then

$x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{b}$. So $\vec{b}$ is a linear combination of the columns of A.

b) As we saw in ⓐ any such x_1, x_2, x_3 is a solution to $A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = b$.

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 1 & 3 & 1 & 3 \\ 2 & 0 & 1 & 5 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 3 & 3 & 3 \\ 0 & 0 & 5 & 5 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 3 & 3 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

3) $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix}$ $\quad x = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ $\quad b = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

a) $AB = \begin{pmatrix} -1/3 + 4/3 & 2/3 - 2/3 \\ -2/3 + 2/3 & 4/3 - 1/3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad BA = \begin{pmatrix} -1/3 + 4/3 & -2/3 + 2/3 \\ 2/3 - 2/3 & 4/3 - 1/3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -1+6 \\ -2+3 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

c) Suppose $Ay = b = Ax$. Then $BAy = BAx$. So $y = Iy = Ix = x$.

d) $C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$. Note that for any $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. $Cx = \begin{pmatrix} x_1 + x_2 \\ 2x_1 + 2x_2 \end{pmatrix}$ $= (x_1 + x_2)\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Now suppose $D = \begin{pmatrix} v_1 & v_2 \end{pmatrix}$. If $CD = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $Cv_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, which is not of the form $\alpha\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. So CD cannot equal I.