Math 340 Spring 2014
Homework #2, Due in Discussion of the Week Feb. 3-7, 2014 — Matrices, Vectors, and Systems of Equations

Remark. Answers should be written in the following format:
A) Result.
B) If possible the name of the method you used.
C) The computation.

1. **Matrices and Vectors.**
   For the following matrices A and vectors $\mathbf{v}$, compute Av.

   (a)
   $$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}.$$

   (b)
   $$A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

   (c)
   $$A = \begin{pmatrix} 1 & -2 & 1 \\ 3 & -4 & 1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

   Note that if $A \in M_{m\times n}$ (A is an $m \times n$ matrix) and $\mathbf{v} \in \mathbb{R}^n$, then $A\mathbf{v} \in \mathbb{R}^m$.

2. **Matrix Arithmetic.**
   Let,
   $$A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 2 \\ 1 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 3 & 5 \\ -4 & 1 & 1 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} -7 \\ 1 \\ 0 \end{pmatrix}, \quad \alpha = 3.$$

   (a) Compute $A\mathbf{u}$, $A\mathbf{v}$, and $A(\mathbf{u}+\mathbf{v})$. Verify that
   $$A\mathbf{u} + A\mathbf{v} = A(\mathbf{u}+\mathbf{v}).$$

   (b) Compute $A(\alpha\mathbf{v})$. Verify that
   $$A(\alpha\mathbf{v}) = \alpha(A\mathbf{v}).$$

   Recall that when multiplying a matrix or a vector by a number, one multiplies each entry of the matrix or vector by that number.

(c) Compute $C = AB$ and $\mathbf{w} = B\mathbf{v}$. Verify that

$$C\mathbf{v} = A\mathbf{w}.$$

In other words $(AB)\mathbf{v} = A(B\mathbf{v})$.

3. **Matrices and Systems of Linear Equations.**

Let

$$A = \begin{pmatrix} 4 & -2 & 7 \\ 1 & 3 & 1 \\ 6 & 4 & 9 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 15 \\ -4 \\ 7 \end{pmatrix}.$$

(a) Find all vectors $\mathbf{x} \in \mathbb{R}^3$ such that $A\mathbf{x} = \mathbf{0}$. Recall that $\mathbf{0}$ is the zero vector whose entries are all zeroes.

(b) Find all vectors $\mathbf{x} \in \mathbb{R}^3$ such that $A\mathbf{x} = \mathbf{b}$.

(c) Denote by $ker(A) = \{\mathbf{x} \in \mathbb{R}^3 | A\mathbf{x} = \mathbf{0}\}$. Show that if $\mathbf{x}, \mathbf{y} \in ker(A)$, and $\alpha \in \mathbb{R}$, then

$$\mathbf{x} + \mathbf{y} \in ker(A), \text{ and } \alpha\mathbf{x} \in ker(A).$$

The set ker(A) is called the kernel of the matrix A. The set of vectors $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ for some $\mathbf{x}$ is called the image of A.

4. Let $A = (a_{ij})$ be a matrix of size $l \times m$ with $a_{ij} \in \mathbb{R}$, for every $1 \leq i \leq l$, $1 \leq j \leq m$, let $B = (b_{jk})$ be a matrix of size $m \times n$ with $b_{jk} \in \mathbb{R}$, for every $1 \leq j \leq m$, $1 \leq k \leq n$, and let

$$v = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_m \end{pmatrix} \in \mathbb{R}^m.$$

(a) Define the vector A times v, in $\mathbb{R}^l$, which is denoted by $A \cdot v$.

(b) Define the matrix $C = A \cdot B$, of size $l \times n$, called the product of A and B.

Remarks

- You are very much encouraged to work with other students. However, submit your work alone.
- The TA and the Lecturer will be happy to help you with the homework. Please visit the office hours.

Good Luck!