Math 340 - Solutions - HW #2

1) a) $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $\quad V = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ $\quad AV = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$

b) $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 1 & 1 \end{pmatrix}$ $\quad V = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\quad AV = \begin{pmatrix} 4 \\ 7 \\ 3 \end{pmatrix}$

c) $A = \begin{pmatrix} 1 & -2 & 1 \\ 3 & -4 & 1 \end{pmatrix}$ $\quad V = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad AV = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

2) $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 2 \\ 1 & 1 & -1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 3 & 5 \\ -4 & 1 & 1 \end{pmatrix}$ $\quad u = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ $\quad V = \begin{pmatrix} -7 \\ 1 \\ 0 \end{pmatrix}$ $\quad \alpha = 3$

a) $Au + AV = \begin{pmatrix} 5 \\ 11 \\ 3 \end{pmatrix} + \begin{pmatrix} -7 \\ -20 \\ -6 \end{pmatrix} = \begin{pmatrix} -2 \\ -9 \\ -3 \end{pmatrix}$ $\quad A\begin{pmatrix} 1-7 \\ 4+1 \\ 2+0 \end{pmatrix} = A\begin{pmatrix} -6 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -9 \\ -3 \end{pmatrix}$

b) $A(\alpha V) = A\begin{pmatrix} -21 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -21 \\ -60 \\ -18 \end{pmatrix}$ $\quad \alpha(AV) = \alpha\begin{pmatrix} -7 \\ -20 \\ -6 \end{pmatrix} = \begin{pmatrix} -21 \\ -60 \\ -18 \end{pmatrix}$

c) $C = AB = \begin{pmatrix} -6 & 2 & 5 \\ -1 & 5 & 16 \\ 7 & 2 & 7 \end{pmatrix}$ $\quad Cv = \begin{pmatrix} 44 \\ 12 \\ -47 \end{pmatrix}$

$w = Bv = \begin{pmatrix} -14 \\ -4 \\ 29 \end{pmatrix}$ $\quad Aw = \begin{pmatrix} 44 \\ 12 \\ -47 \end{pmatrix}$

3) a) $A = \begin{pmatrix} 4 & -2 & 7 \\ 1 & 3 & 1 \\ 6 & 4 & 9 \end{pmatrix}$ $\quad Ax = 0$

$$\left(\begin{array}{ccc|c} 4 & -2 & 7 & 0 \\ 1 & 3 & 1 & 0 \\ 6 & 4 & 9 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 4 & -2 & 7 & 0 \\ 6 & 4 & 9 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & -14 & 3 & 0 \\ 0 & -14 & 3 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 1 & -3/14 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 23/14 & 0 \\ 0 & 1 & -3/14 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{aligned} x_1 &= (-23/14)x_3 \\ x_2 &= (3/14)x_3 \\ x_3 &= x_3 \end{aligned} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t \begin{pmatrix} -23/14 \\ 3/14 \\ 1 \end{pmatrix}$$

b) $Ax = b = \begin{pmatrix} 15 \\ -4 \\ 7 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 4 & -2 & 7 & 15 \\ 1 & 3 & 1 & -4 \\ 6 & 4 & 9 & 7 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 3 & 1 & -4 \\ 4 & -2 & 7 & 15 \\ 6 & 4 & 9 & 7 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 3 & 1 & -4 \\ 0 & -14 & 3 & 31 \\ 0 & -14 & 3 & 31 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 3 & 1 & -4 \\ 0 & 1 & -3/14 & -31/14 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & 23/14 & 37/14 \\ 0 & 1 & -3/14 & -31/14 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{aligned} x_1 &= (37/14) - (23/14)x_3 \\ x_2 &= (-31/14) + (3/14)x_3 \\ x_3 &= x_3 \end{aligned} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 37/14 \\ -31/14 \\ 0 \end{pmatrix} + t \begin{pmatrix} -23/14 \\ 3/14 \\ 0 \end{pmatrix}$$

c)
- Suppose $x, y \in \ker(A)$. Then $Ax = Ay = 0$. So $A(x+y) = Ax + Ay = 0 + 0 = 0$. So $x + y \in \ker(A)$
- Suppose $x \in \ker(A)$, $\alpha \in \mathbb{R}$. Then $Ax = 0$. So $A(\alpha x) = \alpha(Ax) = \alpha \cdot 0 = 0$.