Math 121A Spring 2007

Homework#9 Poles, Laurent series, Residue theorem.

1. Write the Laurent series of f around z_0 and compute its radius of convergence for

   (a) $f(z) = \frac{1}{z(z-1)(z+1)}$ and $z_0 = -1, 0, 1$.

   (b) $f(z) = \frac{z+3}{z^2(z-1)^3(z+1)}$ and $z_0 = -1, 0, 1$.

   (c) $f(z) = \frac{e^z}{z^3}$, $z_0 = 0$.

   (d) $f(z) = \frac{\sin^2 z}{z^3}$, $z_0 = 0$.

2. Compute the residue $res_{z_0}(f)$ for the cases in Problem 1 above.

3. Write the definition (given in class) of a regular point, essential singularity, pole, and order of a pole of a function $f(z)$ at a point z_0. Solve Problems $9, 10, 11$ in M. Boas book, page 682.

4. For the functions of Problem 1 compute $\int_C f(z)dz$ where C is the circle of radius 10 in the plane.

- **Remarks**
  - You are very much encouraged to work with other students. However, submit your work alone.
  - I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good luck!