Math 121A Spring 2007

Homework#8 Integration, Cauchy's theorem.

1. Recall the definition of the integral: Let $f : U \subset \mathbb{C} \to \mathbb{C}$ be a "nice" function and consider a curve $C : [a, b] \to U$. We define the integral of f along C by

$$\int_C f(z)dz \stackrel{def}{=} \int_a^b f(C(t))C'(t)dt.$$

   Show that the Newton-Leibniz formula hold, i.e.,

$$\int_C f'(z)dz = f(C(b)) - f(C(a)).$$

2. Compute the integral $\int_C f(z)dz$ where

   (a) $f(z) = z^3 + 1$ and $C(t) = t + it$, $t \in [0, 1]$.

   (b) $f(z) = z^2$ and C the curve $C(t) = \cos t + i \sin t$, $t \in [0, 2\pi]$.

3. Let f be an analytic function in an open subset $U \subset \mathbb{C}$. Choose two points $z_1, z_2 \in \mathbb{C}$ and two curves C and C' from z_1 to z_2. Show that $\int_C f(z)dz = \int_{C'} f(z)dz$ (Clue: Cauchy's theorem).

4. Do problems $17, 18, 19, 20, 21, 22, 23, 24$ on pages 677-678 in M.L. Boaz book.

- **Remarks**
  - You are very much encouraged to work with other students. However, submit your work alone.
  - I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good luck!