

**Math 121A Spring 2007**  
**Homework#5** Complex functions

1. Use the ratio test to test for convergence of the following series

(a)  $1 + \frac{1+i}{2} + \frac{(1+i)^2}{4} + \frac{(1+i)^3}{8} + \dots + \frac{(1+i)^n}{2^n} + \dots$

(b)  $\sum_{n=0}^{\infty} \left( \frac{1+i}{\sqrt{3}} \right)^n$ .

2. Use the ratio test to find the radius of convergence of the following complex power series

(a)  $1 - z + \frac{z^2}{2} - \frac{z^3}{3} + \frac{z^4}{4} + \dots$

(b)  $\sum_{n=0}^{\infty} \frac{(z+1-i)^n}{3^n n^2}$  (Hint: The interior of a disk of radius 3 and center  $z = -1 + i$ ).

3. Answer the following:

(a) Define the function  $f(z) = \sqrt{z}$  and find real functions  $u(x, y)$ ,  $v(x, y)$  such that  $f = u + iv$ .

(b) Define the function  $g(z) = \ln(z)$  and find real functions  $u(x, y)$ ,  $v(x, y)$  such that  $g = u + iv$ .

4. Consider the function  $e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$ . Show that indeed the series is absolutely converge for every  $z \in \mathbb{C}$ . Prove that  $\frac{d}{dz}(e^z) = e^z$  (Clue: Differentiate term by term).

5. Consider the functions  $\cos(z) = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots$ , and  $\sin(z) = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots$ . Compute  $\frac{d}{dz} \cos(z)$  and  $\frac{d}{dz} \sin(z)$ .

• **Remarks**

- You are very much encouraged to work with other students. However, submit your work alone.
- I will be happy to help you with the homeworks. Please visit me if you want to work with me.

**Good Luck!**