Math 121A Spring 2009

Homework#4 Complex functions

1. Use the ratio test to test for convergence of the following series

   (a) $1 + \frac{1+i}{2} + \frac{(1+i)^2}{4} + \frac{(1+i)^3}{8} + ... + \frac{(1+i)^n}{2^n} + ...$

   (b) $\sum_{n=0}^{\infty} \left(\frac{1+i}{\sqrt{3}}\right)^n$.

2. Use the ratio test to find the radios of convergence of the following complex power series

   (a) $1 - z + \frac{z^2}{2} - \frac{z^3}{3} + \frac{z^4}{4} + ...$

   (b) $\sum_{n=0}^{\infty} \frac{(z+1-i)^n}{3^n n^2}$ (Hint: The interior of a disk of radius 3 and center $z = -1 + i$).

3. Answer the following:

   (a) Define the function $f(z) = \sqrt{z}$ and find real functions $u(x, y)$, $v(x, y)$ such that $f = u + iv$.

   (b) Define the function $g(z) = \ln(z)$ and find real functions $u(x, y)$, $v(x, y)$ such that $g = u + iv$.

4. Consider the function $e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$. Show that indeed the series is absolutely converge for every $z \in \mathbb{C}$. Prove that $\frac{d}{dz}(e^z) = e^z$ (Clue: Differentiate term by term).

5. Consider the functions $\cos(z) = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - ...$, and $\sin(z) = z - \frac{z^3}{3!} + \frac{z^5}{5!} - ...$ Compute $\frac{d}{dz}\cos(z)$ and $\frac{d}{dz}\sin(z)$.

- **Remarks**
  - You are very much encouraged to work with other students. However, submit your work alone.
  - I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good Luck!