Math 121A Spring 2007
Homework#4 $\mathbb{C}$

1. Compute

   (a) $(1+i)^2$, $(1-i)^{-1}$, $\frac{1+i}{1+2i}$.

   (b) Find the polar/standard representation of $1+i$, $2e^{\frac{i\pi}{4}}$, $(1-2i)^{-1}$.

   (c) Draw in the plane the sets $\{z \in \mathbb{C};\ |z+1| = 1\}$, $\{z \in \mathbb{C};\ \mathrm{Im}(z) = \mathrm{Re}(z)\}$.

2. Explain the geometric meaning of multiplication of two complex numbers and draw a picture in the plane (Clue: by De-Moivre's theorem $r_1 e^{i\theta} \cdot\ r_2 e^{i\varphi} = r_1 r_2 e^{i(\theta+\varphi)}$).

3. Find all solutions in $\mathbb{C}$ of the equations $z^2 + 4z + 5 = 0$.

4. Find all complex numbers that satisfy $z^4 = 1$. Draw the results in the plane (Clue: De-Moivre's theorem).

5. Recall the definition $e^{i\theta} = \cos\theta + i\sin\theta$. What trigonometric formulas you get from equating both sides of the identity $e^{i\theta} \cdot\ e^{i\varphi} = e^{i(\theta+\varphi)}$?

- **Remarks**
  - You are very much encouraged to work with other students. However, submit your work alone.
  - I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good Luck!