

Math 121A Spring 2007
Homework#3 Linear Algebra

1. Consider the vector space $V = \mathbb{R}^2$ and the basis $\mathcal{B} = \{(1, 1), (1, 2)\}$ and $\mathcal{C} = \{(1, 0), (1, 1)\}$. Compute the transition matrix $T = T_{\mathcal{C}, \mathcal{B}}$. Suppose $v \in V$ with $[v]_{\mathcal{B}} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Compute $[v]_{\mathcal{C}}$.

2. Consider the vector space $V = \{ax^2 + bx + c; a, b, c \in \mathbb{R}\}$ of all the polynomial of degree ≤ 2 . Consider the linear operator $L : V \rightarrow V$ given by $L = \partial^2 + \partial + I$. For example $L(x^2) = x^2 + 2x + 2$. Compute $\det(L)$ and $\text{Tr}(L)$ (Clue: $\det$ and Tr of an operator are independent of any choice of a basis).

3. Compute eigenvalues and eigenvectors for the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ and find a matrix B such that $BAB^{-1} = D$ diagonal.

4. Compute

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{29}.$$

(Clue: Problem 3).

5. * (Student's question from office hour) Consider the vector space $V = \mathbb{R}^2$ and let $R_\theta : V \rightarrow V$ be the transformation of rotation of the plane by angle θ around 0.

(a) Show that R_θ is linear.

(b) Consider the standard basis $\mathcal{B} = \{(1, 0), (0, 1)\}$ of V . Compute the matrix $[R_\theta]_{\mathcal{B}}$. Compute its determinant.

(c) Consider the standard inner-product $\langle u, v \rangle$ on V . Show that it is invariant under the transformation R_θ , i.e., show that for any two vectors $u, v \in V$ we have $\langle R_\theta u, R_\theta v \rangle = \langle u, v \rangle$ (Clue: use the fact that $\langle u, v \rangle = \cos \alpha(u, v) \cdot \|u\| \|v\|$, where $\alpha(u, v)$ is the angle between u and v , and $\|u\|$ denote the length of u).

• **Remarks**

- You are very much encouraged to work with other students. However, submit your work alone.
- I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good Luck!