Math 121A Spring 2009
Homework#2 Linear Algebra

1. Consider the vector space $V = \mathbb{R}^2$ and the basis $\mathcal{B} =\{(1,1),(1,2)\}$ and $\mathcal{C} = \{(1,0),(1,1)\}$. Compute the transition matrix $T = T_{\mathcal{C},\mathcal{B}}$. Suppose $v \in V$ with $[v]_{\mathcal{B}} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Compute $[v]_{\mathcal{C}}$.

2. Consider the vector space $V = \{ax^2 + bx + c;\, a,b,c \in \mathbb{R}\}$ of all the polynomial of degree ≤ 2. Consider the linear operator $L : V \to V$ given by $L = \partial^2 + \partial + I$. For example $L(x^2) = x^2 + 2x + 2$. Compute $\det(L)$ and $Tr(L)$ (Clue: det and Tr of an operator are independent of any choice of a basis).

3. Compute eigenvalues and eigenvectors for the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ and find a matrix B such that $BAB^{-1} = D$ diagonal.

4. Compute
$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{29}.$$
(Clue: Problem 3).

5. * (Student's question from office hour) Consider the vector space $V = \mathbb{R}^2$ and let $R_\theta : V \to V$ be the transformation of rotation of the plane by angle θ around 0.
   (a) Show that R_θ is linear.
   (b) Consider the standard basis $\mathcal{B} = \{(1,0),(0,1)\}$ of V. Compute the matrix $[R_\theta]_{\mathcal{B}}$. Compute its determinant.
   (c) Consider the standard inner-product $\langle u, v\rangle$ on V. Show that it is invariant under the transformation R_θ, i.e., show that for any two vectors $u, v \in V$ we have $\langle R_\theta u, R_\theta v\rangle = \langle u, v\rangle$ (Clue: use the fact that $\langle u, v\rangle = \cos\alpha(u,v) \cdot \|u\|\,\|v\|$, where $\alpha(u,v)$ is the angle between u and v, and $\|u\|$ denote the length of u).

- **Remarks**
  - You are very much encouraged to work with other students. However, submit your work alone.
  - I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good Luck!