

Math 121A Spring 2007
Homework#2

1. Let $A \in M_3(\mathbb{R})$. Denote by $v_1, v_2, v_3 \in \mathbb{R}^3$ the rows of A . Show that $\det(A) = \pm \text{Vol}(v_1, v_2, v_3)$ the volume of the parallelepiped defined by v_1, v_2, v_3 .
2. In M. L. Boaz, page 123 problems 13, 15.
3. Let $M_2(\mathbb{R})$ be the algebra of all 2×2 matrices over the field \mathbb{R} . Consider the subset $V \subset M_2(\mathbb{R})$ given by

$$V = \{A; \text{tr}(A) = 0\}.$$

Show that V is a vector space, i.e., show that if $A, B \in V$ then $A + B \in V$ and also if $\alpha \in \mathbb{R}$ then $\alpha A \in V$.

4. Consider the vector space V from problem 3. Show that the set

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right\}$$

form a base to V . Compute $\dim(V) = ?$.

5. Consider the vector space V and the base \mathcal{B} as in problem 4. Compute the coordinate vector

$$\left[\begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix} \right]_{\mathcal{B}} = ?$$

• **Remarks**

- You are very much encouraged to work with other students. However, submit your work alone.
- I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good Luck!