Math 121A Spring 2009
Homework#10 DFT and Motions of the Plane

1. Discrete Fourier transform.

   (a) For a function $f : \mathbb{Z}_N \to \mathbb{C}$ define its Fourier transform $\widehat{f} : \mathbb{Z}_N \to \mathbb{C}$.

   (b) For two function $f, g : \mathbb{Z}_N \to \mathbb{C}$ define the convolution $f * g : \mathbb{Z}_N \to \mathbb{C}$ and prove the identity $\widehat{f * g} = \widehat{f} \cdot \widehat{g}$. Clue: $\{\delta_a\}$.

   (c) Suppose A and B are two polynomials of degree d. Explain how to compute the multiplication $C = AB$ in a approximately $N \log(N)$ operations, where $N = 2d$. You should assume the Cooley-Tukey FFT algorithm.

2. The group of motions of the plane.

   (a) Define when a map $m : \mathbb{R}^2 \to \mathbb{R}^2$ is called a *motion*. Write down the classification theorem for the group M of all motion of the plane.

   (b) Let l_1 and l_2 be two lines in the plane which intersect in one point $p = l_1 \cap l_2$ and with angle α between them. Compute explicitly the motion $m = r_{l_2} \circ r_{l_1}$ (Clue: Use the classification theorem).

   (c) Define when a linear map $A : \mathbb{R}^2 \to \mathbb{R}^2$ is called *orthogonal. Show that the set* O_2 of all the orthogonal maps is a subgroup of M. Show that $\det(A) = \pm 1$ for every $A \in O_2$. Show that if $A \in O_2$ then A is a rotation R_θ or rotation composed with the standard reflection $r : (x, y) \mapsto (x, -y)$.

- **Remarks**

  - You are very much encouraged to work with other students. However, submit your work alone.
  - I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good luck!