Math 113 Spring 2007
HW#8: Equivalence relations, Partitions and Cosets

1. Let $\varphi : G \to G'$ be homomorphism of groups. Show that φ is injective if an only if $\ker(\varphi) = 1$.

2. Write the definition of an equivalence relation on a set X. Solve the following problems from Artin's Book: 5.2, 5.5, 8.

3. Let G be a group and define the conjugation relation $\sim$ on G by: $x \sim y$ if there exist $g \in G$ so that $y = gxg^{-1}$. Show that $\sim$ is an equivalence relation on G.

4. Let $n > 1$ be an integer and consider the subgroup $n\mathbb{Z} \subset \mathbb{Z}$. Describe explicitly the set $\mathbb{Z}/n\,\mathbb{Z}$ of left cosets and compute the index $[\mathbb{Z} : n\mathbb{Z}]$. Show that there is a natural bijection between $\mathbb{Z}/n\,\mathbb{Z}$ and the set $\mathbb{Z}_n = \{0, 1, ..., n-1\}$.

5. Let G be a group of order 8, i.e., $\#G = 8$. Explain why there is no element $g \in G$ with order 3. What might be the order of g?

6. Prove that every group G such that $\#G = p^k$, where p is a prime number and $k > 0$ an integer, contains a subgroup of order p.

- You are very much encouraged to work with other students. However, submit your work alone.
- I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good Luck!