**Math 113 Spring 2007**
**HW#5: Subgroups.**

1. For each of the following decide if it is a subgroup.

    (a) The subset $H \subset S_5 = Aut(\{1, ...5\})$ with $H = \{\sigma \in S_5;\ \sigma(1) = 1\}$.

    (b) The subset $\mathbb{R}^\times = \mathbb{R} - \{0\}$ in $\mathbb{C}^\times = \mathbb{C} - \{0\}$ with multiplication.

    (c) The subset of positive real numbers in $\mathbb{R}^\times$ with multiplication.

    (d) The subset $\mu_n = \{z \in \mathbb{C}^\times; z^n = 1\}$ in $\mathbb{C}^\times$.

    (e) The subset $T \subset GL_2(\mathbb{R})$ with $T = \{\begin{pmatrix} a & \\ & b \end{pmatrix};\ ab \neq 0\}$.

2. Let $G$ be a group and $X$ a set. An action of $G$ on $X$ is an assignment for every $g \in G$ and every $x \in X$ an element $g \cdot x \in X$ such that $(gg') \cdot x = g \cdot (g' \cdot x)$ and $1_G \cdot x = x$. Show that

    (a) If $G$ act on $X$ then for every $g \in G$ the induced map $\alpha_g : X \to X$ given by $\alpha_g(x) = g \cdot x$ is a bijection.

    (b) If $G$ act on $X$ then for every subset $Y \subset X$ the *stabilizer* $H = Stab_G(Y) = \{g \in G;\ g \cdot y \in Y$ for every $y \in Y\}$ is a subgroup of $G$.

    (c) Use the notion of stabilizer to show that the set $T = \{\begin{pmatrix} a & \\ & b \end{pmatrix};\ ab \neq 0\} \subset GL_2(\mathbb{R})$ is a subgroup (Clue: $G = GL_2(\mathbb{R})$ with its standard action on $\mathbb{R}^2$ and $Y = \{(x, y)\,;\ x = 0 \text{ or } y = 0\}$).

3.

4.

5.