

Math 113 Spring 2007
HW#10: Symmetry

1. Consider the plane $P = \mathbb{R}^2$ with its standard distance $d(p, q)$. Define when a map $m : P \rightarrow P$ is a motion.
2. Show that the set M of all motions of the plane is a group under the operation of composition.
3. Using the theorem that classify all possible motions of the plane show that
 - (a) The composition $r_{l_2} \circ r_{l_1}$ of two reflections with respect to two different intersecting lines is a rotation $R_{\theta, p}$ with angle θ around $p = l_1 \cap l_2$.
 - (b) The composition $r_{l_2} \circ r_{l_1}$ of two reflections with respect to two different parallel lines is a translation t_a . Compute explicitly the vector $a \in \mathbb{R}^2$.
4. Explain, using the classification theorem for M , for each of the motions below if it is a translation, a rotation, a reflection or a glide reflection;
 - (a) $m = t_a \circ R_{\theta, p}$ a rotation composed with a translation.
 - (b) $m = R_{\eta, q} \circ R_{\theta, p}$ composition of two rotations.
 - (c) $m = r_l \circ R_{\theta, p}$ composition of rotation and a reflection.
- You are very much encouraged to work with other students. However, submit your work alone.
 - I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good Luck!