Math 113 Spring 2007
HW#1: How to deliver a function?

1. Write the *sum* and *multiplication* tables for the field $\mathbb{F}_3 = \{0, 1, 2\}$.

2. Show that all the solutions in $\mathbb{C}$ for the equation $z^3 = 1$ are given by $e^{\frac{2\pi i a}{3}}$, $a = 0, 1, 2$. Draw them in the plane.

3. Let $f : \mathbb{F}_3 \to \mathbb{C}^\times$ be a function that satisfy the identities $f(0) = 1$ and $f(x + y) = f(x)f(y)$. Show that $f(x) = e_a(x) = e^{\frac{2\pi i a x}{3}}$ for some $a \in \mathbb{F}_3$. (Clue: $1 + 1 + 1 = 0$ and $f(1 + 1 + 1) = f(1)^3$. Then use Problem 2 above).

4. Using Problem 3 above, explain how to "deliver by the phone" the function e_1 without mentioning an explicit formula.

5. ★ Consider the vector space $V = \mathbb{C}(\mathbb{F}_3)$ and for every $y \in \mathbb{F}_3$ define the operator $R_y : V \to V$ by the formula

$$[R_y f](x) = f(x + y).$$

Explain that the solutions for the eigenfunction problem

$$\begin{aligned} R_y f &= \lambda(y) f, \\ f(0) &= 1, \end{aligned}$$

are exactly the functions e_0, e_1, e_2.

- **Remarks**
  - You are very much encouraged to work with other students. However, submit your work alone.
  - I will be happy to help you with the homeworks. Please visit me if you want to work with me.

Good Luck!!