**Math 110 Fall 2008**

**HW#2: Definition of a vector space and subspace**

1. Do Exercises 10 - 14 on pages 14-15.

2. Let $F = \{0\}$ with operations $+$ and $\cdot$ defined by $0+0=0$, $0 \cdot 0 = 0$. Now, assume that in the definition of a vector space we use this specific $F$. Show that any "vector space" $V$ over $F$ is trivial, i.e., $V = \{0_V\}$.

3. Let $V$ and $W$ be vector space over a field $F$. Define a new set $V \times W = \{(v,w);\ v \in V,\ w \in W\}$. Show that with the operations $(v,w)+(v',w') = (v+v', w+w')$ and $\alpha(v,w) = (\alpha v, \alpha w)$, and $0_{V\times W} = (0_V, 0_W)$, we have that $V \times W$ is a vector space over $F$.

4. Let $V$ be a vector space over $F$ and $W \subset V$ a subset. Show that $W$ is a vector space over $F$ (with the operations $+$ and multiplication by scalars on $V$) if and only if the following conditions hold: $0_V \in W$, $w_1 + w_2 \in W$ for every $w_1, w_2 \in W$, $\alpha w \in W$ for every $\alpha \in F$ and $w \in W$.

5. Let $V, W$ be vector spaces over a field $F$. Show that $V \cap W$ is a subspace of $V$.

- **Remark** You are very much encouraged to work with other students. However, submit your work alone.

**Good Luck!!**