Math 110 Fall 2008
HW#1: Fields

1. Read the definition of a field and the examples in Appendix C.

2. Prove Theorem C.1 on page 554 of the Book. Prove the Corollary on page 554.

3. Prove Theorem C.2 on page 555. Prove the Corollary on page 555.

4. Denote by $\mathbb{F}_p$ the set $\{0, 1, ..., p-1\}$, where p is a prime number. It can be shown that $\mathbb{F}_p$ is a field where $+$ and $\cdot$ are defined modulo p. Write the multiplication table and the addition table in $\mathbb{F}_5$.

5. Read the definition of the field $\mathbb{C}$ of complex numbers in page 556. We will return to study and use this field later. One of the importance of this field is that every polynomial $P(x) \in \mathbb{C}[x]$ has a root. As you know this is not true if we replace $\mathbb{C}$ by $\mathbb{R}$, e.g., $P(x) = x^2 + 1$.

- **Remark** You are very much encouraged to work with other students. However, submit your work alone.

Good Luck!!