

Math 104, Spring 09
Homework#9: Continuous functions

1. Do exercises 1.,2.,3.,4.,5.,6.,14. page 98 in the book.
2. Prove that a function $f : X \rightarrow Y$ (X and Y are metric spaces) is continuous if and only if for every open subset $V \subset Y$ we have that the *pre-image* $f^{-1}(V) = \{x \in X \text{ with } f(x) \in V\}$ is open in X (pay attention: f need not be invertible!).
3. Let X and Y be two sets and $f : X \rightarrow Y$ be a function. For a subset $S \subset X$ define its *image* $f(S) = \{f(s); s \in S\}$. Explain what is the relation between $f^{-1}(f(S))$ and S ? And what is the relation between $f(f^{-1}(T))$ and T for a subset $T \subset Y$. Also assume $\bigcup_{\alpha} T_{\alpha} \subset Y$ and explain the relation between $f^{-1}(\bigcup_{\alpha} T_{\alpha})$ and $\bigcup_{\alpha} f^{-1}(T_{\alpha})$ and for $\bigcup_{\alpha} S_{\alpha} \subset X$ explain the relation between $f(\bigcup_{\alpha} S_{\alpha})$ and $\bigcup_{\alpha} f(S_{\alpha})$?
4. Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous. Prove that f is determined from its values on $\mathbb{Q}$.
5. Consider the function $f : [0, 1] \rightarrow \mathbb{R}$ given by $f(x) = x^2$. Given $\epsilon > 0$ find an explicit $\delta > 0$ so that for every $x, y \in [0, 1]$ with $|x - y| < \delta$ we have $|f(x) - f(y)| < \epsilon$.

Good luck!!