Math 104, Fall 07
Homework#:7 Series

1. Write a proof that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log(n))^{\alpha}}$ converge if $\alpha > 1$ and diverge for $\alpha \leq 1$ (You can use the proof on page 62 in the book).

2. Find whether the following series diverge or converge:

   (a) $\sum_{n=1}^{\infty} \frac{n}{n^2+11}$.

   (b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+9}}$.

3. Do exercises 6,7,8 page 78 in the book.

4. Let (a_n) be a sequence of real numbers. Let $\alpha = \limsup a_n$. Show that if $\beta > \alpha$ then there is an a natural number N such that $a_n < \beta$ for every $n \geq N$ (Clue: This is more or less follow from the definition on page 56 in the book).

5. Use the ratio test, root test or any other method to test for convergence

   (a) $\sum_{n=1}^{\infty} \frac{n!}{100^n}$.

   (b) $\sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!}$.

   (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$.

Good luck!!