

Math 104, Spring 09
Homework#5 Sequences

1. Give an example of a metric space which is not complete, i.e., in which there exists a Cauchy sequence which is not converge.
2. Let (X, d) be a compact metric space. Show that X is complete.
3. Let X be a complete metric space. Show that every closed subset $F \subset X$ is also complete, i.e., show that if $(x_n) \subset F$ is a Cauchy sequence then it has a limit L in F .
4. Consider the sequence $x_{n+1} = \sqrt{2 + x_n}$ with $x_1 = 1$. Show that this sequence converge and compute its limit.
5. Show that a convergent sequence in a metric space is bounded.

Good luck!!