

Math 104, Fall 07
Homework#:5 Sequences

1. Show using only the definition that

(a) $\frac{1}{n+1} \rightarrow 0$, as $n \rightarrow \infty$.

(b) $\frac{1}{n^2} \rightarrow 0$, as $n \rightarrow \infty$.

(c) $1 + (-1)^n$ diverge as $n \rightarrow \infty$.

2. Suppose (a_n) and (b_n) are sequences of complex or real numbers. Assume that $a_n \rightarrow a$ and $b_n \rightarrow b$. Show using the definition of convergence that

(a) The sequence $(a_n + b_n)$ converge to $a + b$.

(b) If α is a fixed number then the sequence $(\alpha \cdot a_n)$ converge to $\alpha \cdot a$.

3. Problems 2, 3, page 78 in the book.

4. Prove from the definition that for $\alpha > 0$, $\frac{1}{n^\alpha} \rightarrow 0$ as $n \rightarrow \infty$.

5. Prove the binomial theorem $(a+b)^n = \dots$ (You should also write the correct statement).

6. In class we proved that for every real number $x \geq 1$ we have $\sqrt[n]{x} \rightarrow 1$ as $n \rightarrow \infty$. Prove this also in the case $0 < x < 1$.

Good luck!!