Math 104, Spring 09
Homework#:4 Sequences

1. Show using only the definition that

   (a) $\frac{1}{n+1} \to 0$, as $n \to \infty$.

   (b) $\frac{1}{n^2} \to 0$, as $n \to \infty$.

   (c) $1 + (-1)^n$ diverge as $n \to \infty$.

2. Suppose (a_n) and (b_n) are sequences of complex or real numbers. Assume that $a_n \to a$ and $b_n \to b$. Show using the definition of convergence that

   (a) The sequence $(a_n + b_n)$ converge to $a + b$.

   (b) If α is a fixed number then the sequence $(\alpha \cdot a_n)$ converge to $\alpha \cdot a$.

3. Problems $2, 3$, page 78 in the book.

4. Prove from the definition that for $\alpha > 0$, $\frac{1}{n^\alpha} \to 0$ as $n \to \infty$.

5. Prove the binomial theorem $(a+b)^n = ...$(You should also write the correct statement).

6. In class we proved that for every real number $x \geq 1$ $\sqrt[n]{x} \to 1$ as $n \to \infty$. Prove this also in the case $0 < x < 1$.

Good luck!!