

Math 104, Fall 07
Homework#4: Compact subsets

1. Give an example for a bounded set in $\mathbb{R}$ which is not compact. Give an example for a closed subset in $\mathbb{R}$ which is not compact.
2. Write down examples of compact and non-compact subsets in $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$.
3. Show that the subset $E = [0, 1) \subset \mathbb{R}$ is not compact. Find an explicit cover of E with no finite sub-cover.
4. Proof that if E is an infinite subset of a compact subset K then E has a limit point in K . Here is a direction: If no point of K is a limit point of E then each $x \in K$ have a neighborhood V_x which contains at most one point of E . Explain why the collection $\{V_x; x \in K\}$ is a cover of K for which no subcollection can cover E and in particular can not cover K .

Good luck!!