

Math 104, Spring 09
Homework#3: Topology in metric spaces

1. Write down several examples of open and closed subset of $\mathbb{R}$ and $\mathbb{R}^2$. Give examples also for subsets which are not closed and not open.
2. Recall the definition of a limit point of a subset E of a metric space X . Let E' be the set of all limit points of E . Show that E' is closed. (Clue: consider the complement of E').
3. Explain why a finite set of points in a metric space has no limit point.
4. Give an example of a subset E in a metric space X such that $E' \subsetneq E$.
5. We say that a subset E of a metric space X is *dense* in X if $X = E \cup E'$. Show that $\mathbb{Q}$ is dense in $\mathbb{R}$.
6. Let (X, d) be a metric space and $E \subset X$. A point $x \in E$ is called an *interior point* of E if there exist a neighborhood U of x such that $U \subset E$. Denote by E° the set of all interior points of E . Show that E is open if and only if $E = E^\circ$.

Good luck!!