

Math 104, Spring 09
Homework#2: Euclidean spaces and Metric spaces

1. Problems 17, 18, 19 page 23.
2. Problem 11, page 44.
3. Draw the following sets in \mathbb{R}^2 and \mathbb{R}^3 :
 - (a) The open ball $B(0, 1)$.
 - (b) The closed ball $\overline{B(0, 1)}$.
 - (c) The boundary $\partial B(0, 1) = \{x \in \mathbb{R}^n; |x| = 1\}$.
4. Let X be a set and consider the function d on $X \times X$ defined by $d(x, y) = 1$ if $x \neq y$ and $d(x, x) = 0$. Prove that d is a metric on X and compute the open ball $B(x, 1)$ and the closed ball $\overline{B(x, 1)}$ around a general point $x \in X$.
5. Show that any ball (open or closed) in a Euclidean space \mathbb{R}^n is convex set (Clue: triangle inequality).

Good luck!!