Math 104, Fall 07

Homework#2: Euclidean spaces and Metric spaces

1. Problems 17, 18, 19 page 23.

2. Problem 11, page 44.

3. Draw the following sets in $\mathbb{R}^2$ and $\mathbb{R}^3$:

   (a) The open ball $B(0,1)$.

   (b) The closed ball $\overline{B(0,1)}$.

   (c) The boundary $\partial B(0,1) = \{x \in \mathbb{R}^n;\ |x| = 1\}$.

4. Let X be a set and consider the function d on $X \times X$ defined by $d(x,y) = 1$ if $x \neq y$ and $d(x,x) = 0$. Prove that d is a metric on X and compute the open ball $B(x,1)$ and the closed ball $\overline{B(x,1)}$ around a general point $x \in X$.

5. Show that any ball (open or closed) in a Euclidean space $\mathbb{R}^n$ is convex set (Clue: triangle inequality).

Good luck!!