Math 104, Fall 07
Homework#1: $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$

1. Show that $\sqrt{3} \notin \mathbb{Q}$.
2. Let $(S, <)$ be an ordered set and $E \subset S$ a set which is bounded below. Define the <u>infimum</u> α of E which is denoted by $\alpha = \inf E$. Show that there exist at most one such α.
3. Read the section "Fields", pages 5-8 in Rudin's book, and submit a proof of Proposition 1.18.
4. Recall that if x is a positive real number and n is a positive integer then there exist a unique positive real number y with $y^n = x$. The number y will be denoted by $\sqrt[n]{x}$. Show that if a and b are two positive real numbers and n is a positive integer then $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$. (Clue: the uniqueness of the n'th root).
5. Proof by induction on n that $|z_1 + ... + z_n| \leq |z_1| + ... + |z_n|$ for $z_{1,...,}z_n \in \mathbb{C}$. (Assume that the case $n = 2$ was proved in class).
6. Solve problem 14 on page 23 of Rudin's book. (Clue: $|z|^2 = z \cdot \overline{z}$).

Good luck!!