Math 104, Fall 07

Homework#12: Taylor's theorem, integration

1. Use Taylor's theorem to compute $\sqrt[3]{90}$ with approximation which is better then 0.001.
2. Compute, using only the definition of integral using Riemann sums, the integral $\int\limits_{-1}^{1} x^2 dx$.
3. Prove the theorem on change of variable in integration. **Theorem:** suppose $g : [A.B] \to [a,b]$ a bijection with $g'(x) > 0$ for every $x \in (a,b)$. Suppose $f : [a,b] \to \mathbb{R}$ integrable. Then $\int\limits_{A}^{B} f(g(x))g'(x)dx = \int\limits_{a}^{b} (f(u)du$.
4. Consider the function on $[-2,2]$ given by $f(t) = e^{t^2}$. Use the primitive function theorem to find a primitive F of f on $[-2,2]$. Compute $F'(0)$.
5. Compute $F'(x)$ where $F(x) = \int\limits_{0}^{\sin x} \cos(t)dt$.
6. Compute $\int\limits_{0}^{1} x^2 e^x dx$. Explain the theoretical theorems that you use in order to do the computations. Explain why the conditions hold true so that you are allowed to use the theorems.
7. Recall the chain rule and use it to prove the integration by part theorem.
8. Show that if f, g are integrable on $[a,b]$ so does there multiplication $f \cdot g$.

Good luck!!