**Math 104, Fall 07**

**Homework#11:** Connectedness, differentiation

1. State and prove the theorem that a continuos function maps connected set to a connected set (Theorem 4.22 on page 93 of the book).

2. Let $I = [a, b] \subset \mathbb{R}$ be a closed interval with $a < b$. Consider a continuos function $f : I \to I$. Show that there exist a point $x \in I$ such that $f(x) = x$. (Clue: Cauchy's mean value theorem).

3. Consider two differentiable (has a derivative at every point) functions $f, g : \mathbb{R} \to \mathbb{R}$. Show that $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$ for every $x \in \mathbb{R}$. In addition, if $g(x) \neq 0$ then $(\frac{f}{g})'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$.

4. Use the Lagrange mean value theorem: $\frac{f(b)-f(a)}{b-a} = f'(c)$ for some $a \leq c \leq b$ to compute approximation of $\sqrt{27}$.

**Good luck!!**