**Math 104, Fall 07**

**Homework#10:** Continuous functions

1. Do exercises 1.,2.,3.,4.,5.,6.,14. page 98 in the book.

2. Prove that a function $f : X \to Y$ ($X$ and $Y$ are metric spaces) is continuos if an only if for every open subset $V \subset Y$ we have that the *pre-image* $f^{-1}(V) = \{x \in X \text{ with } f(x) \in V\}$ is open in $X$ (pay attention: $f$ need not be invertible!).

3. Let $X$ and $Y$ be two sets and $f : X \to Y$ be a function. For a subset $S \subset X$ define its *image* $f(S) = \{f(s);\ s \in S\}$. Explain what is the relation between $f^{-1}(f(S))$ and $S$? And what is the relation between $f(f^{-1}(T))$ and $T$ for a subset $T \subset Y$. Also assume $\bigcup\limits_{\alpha} T_\alpha \subset Y$ and explain the relation between $f^{-1}(\bigcup\limits_{\alpha} T_\alpha)$ and $\bigcup\limits_{\alpha} f^{-1}(T_\alpha)$ and for $\bigcup\limits_{\alpha} S_\alpha \subset X$ explain the relation between $f(\bigcup\limits_{\alpha} S_\alpha)$ and $f \bigcup\limits_{\alpha} f(S_\alpha)$?

4. Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuos. Prove that $f$ is determined from its values on $\mathbb{Q}$.

5. Consider the function $f : [0, 1] \to \mathbb{R}$ given by $f(x) = x^2$. Given $\epsilon > 0$ find an explicit $\delta > 0$ so that for every $x, y \in [0, 1]$ with $|x - y| < \delta$ we have $|f(x) - f(y)| < \epsilon$.

**Good luck!!**