Title: Dimensionality and patterns with curvature

Abstract: Hausdorff dimension is a notion of size ubiquitous in geometric measure theory. A set of large Hausdorff dimension contains many points, so it is natural to expect that it should contain specific configurations of interest. Yet many existing results in the literature point to the contrary. In particular, there exist full-dimensional sets K in the plane with the property that if a point (x_1, x_2) is in K, then no point of the form $(x_1, x_2 + t)$ lies in K, for any $t \neq 0$.

A recent result of Kuca, Orponen and Sahlsten shows that *every* planar set of Hausdorff dimension sufficiently close to 2 contains a two-point configuration of the form $(x_1, x_2) + \{(0, 0), (t, t^2)\}$ for some $t \neq 0$. This suggests that sets of sufficiently large Hausdorff dimension may contain patterns with "curvature", suitably interpreted. In joint work with Benjamin Bruce, we obtain a characterization of smooth functions $\Phi : \mathbb{R} \to \mathbb{R}^d$ such that every set of sufficiently high Hausdorff dimension in d-dimensional Euclidean space contains a two point configuration of the form $\{x, x + \Phi(t)\}$, for some t with $\Phi(t) \neq 0$.