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Approximation by N-term trigonometric polynomials and greedy algorithms

Abstract:

Given a Fourier series $f(x) = \sum_{n\in\mathbb{Z}} \hat{f}(n)e^{inx}$ in $L^p(\mathbb{T})$, a natural question in approximation theory is to find a frequency set $A \subset \mathbb{Z}$ with $|A| \leq N$ which minimizes

$$\|f - \sum_{n\in A} \hat{f}(n)e^{inx}\|_p.$$

When $p = 2$, this occurs when A corresponds to the largest coefficients $|\hat{f}(n)|$. When $p \neq 2$, it is an open question how to find such a constructive procedure.

In this talk we discuss this problem using a variation of the above procedure, called Weak Chebyshev Greedy Algorithm (WCGA), $\mathscr{G}_N(f)$, which gives near optimal results when $p > 2$. More precisely, if Σ_N is the set of N-term trigonometric polynomials, then

$$\|f - \mathscr{G}_{\phi(N)}(f)\|_p \lesssim \text{dist } (f, \Sigma_N), \tag{1}$$

with $\phi(N) = O(N \log N)$.

The WCGA is a generalization to Banach spaces of the popular Orthogonal Matching Pursuit (OMP) from signal processing. The above result is a special case of a deep theorem of Temlyakov, which establishes conditions of a basis and a Banach space so that (1) holds for a suitable $\phi(N)$.

In this talk we present an improvement of the result of Temlyakov which is near optimal for the spaces $L^p(\log L)^\alpha$. In particular, for the trigonometric system in $L^2(\log L)^\alpha$, with $\alpha > 0$, we show that it suffices with $\phi(N) = O(N \log\log N)$ iterations.

References

[1] G. Garrigós. The WCGA in $L^p(\log L)^\alpha$ spaces. Preprint 2022, available in `webs.um.es/gustavo.garrigos`