MAXIMAL AVERAGES ALONG HYPERSURFACES: A new conjecture and almost complete answers in $\mathbb{R}^3$

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The study of L^p-estimates for maximal averages $\mathcal{M}_S$ associated to isotropic dilates of a given, smooth hypersurface S in Euclidean space originated from E.M. Stein's seminal work on dimension free estimates for the Hardy-Littlewood maximal operator, in which he had studied the spherical maximal function.

By localization, one can reduce to studying small surface-patches S near a given point x^0. Denoting by p_c the minimal Lebesgue exponent such that $\mathcal{M}_S$ is L^p-bounded for $p > p_c$, I shall first explain a new "geometric" conjecture on how the critical exponent p_c might be determined by means of a geometric measure theoretic condition, which measures in some sense the order of contact of arbitrary ellipsoids with S.

The main part of the talk will then focus on hypersurfaces in $\mathbb{R}^3$, for which we are able by now to identify p_c for almost all analytic surfaces (with the exception of a small subclass $\mathcal{A}^e$ of surfaces exhibiting singularities of type $\mathcal{A}$ at x^0), by means of quantities which can be determined from associated Newton polyhedra. Besides the well-known notion of height at x^0, a new quantity, which we call the effective multiplicity, turns out to play a crucial role here.

Our recent results lead in particular to a proof of the "geometric" conjecture for all analytic 2-surfaces which are not of exceptional class $\mathcal{A}^e$, as well as the proof of a conjecture by Iosevich-Sawyer-Seeger for arbitrary analytic 2-surfaces.