Math 121A: Midterm information

- The first class midterm will take place in class on Wednesday, February 27th from 3:10pm–4pm.
- The exam will cover everything in class up to and including the lecture on Wednesday, February 20th. This corresponds to Chapters 1–5 in Boas, with several minor exceptions: §3.13 (groups) and §5.4–5.6 (surface integration and change of variables) will not be covered.
- The exam is closed book – no textbooks, notebooks, or calculators allowed.
- There will be four questions, each of which will be graded out of ten points.

Sample midterm questions

1. By using appropriate series tests, determine whether

$$\sum_{n=0}^{\infty} \frac{1}{4^n + (1/2)^n}, \qquad \sum_{n=1}^{\infty} \sin \frac{n\pi}{4}, \qquad \sum_{n=2}^{\infty} \frac{1}{n \log n}$$

converge or diverge.

2. (a) Calculate the exact interval of convergence of

$$\sum_{n=0}^{\infty} \frac{y^n}{3^n}.$$

   (b) Sketch the function $f(x) = 3 + (x^2 - 1)(x^2 - 4)$ over the range $-3 < x < 3$.

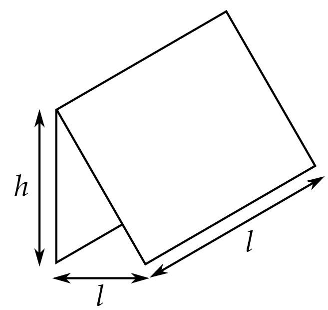
   (c) Determine the exact ranges of convergence of

$$\sum_{n=0}^{\infty} \frac{f(x)^n}{3^n}.$$

3. Consider a right-angled triangle with vertices at $(0, 0)$, $(0, 1)$, and $(1, 0)$, with non-uniform density $\rho(x, y) = y$. Find its center of mass.

4. Find the minimum and maximum values of the function $f(x, y) = (x - 2y)^2 - x$ in the square $|x| \leq 1$, $|y| \leq 1$.

5. *(The algebra on this question is a little more complicated than would expected on the midterm, but it is good practice.)* Consider an asymmetric tent design of length l that is comprised of a vertical section of height h, connected to a diagonal section, as shown in the diagram below.



The volume of the tent is $hl^2/2$. For a fixed area of tent material A, find the values of h and l that maximise the tent's volume.