Math 121A: Midterm 1

1. The function

$$f(x, y, z) = x^2 + 3y^2 + 5z^2 + 2xy + 4yz - 4x - 2z$$

has one minimum point. Find its location.

2. (a) Calculate the derivative of $f(x) = \log(\log(\log(x)))$.

   (b) By using an appropriate series test, determine whether

$$\sum_{n=3}^{\infty} \frac{1}{n \log(n) \log(\log(n))}$$

   converges or diverges.

   (c) By using an appropriate series test, determine whether

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{n \log(n) \log(\log(n))}$$

   converges or diverges.

3. By using Lagrange multipliers, find the smallest possible surface area (including both ends) of a cylinder with volume V.

4. (a) By considering appropriate powers of $e^{i\theta} = \cos\theta + i \sin\theta$ or otherwise, determine an expression for $\sin^3\theta$ as a linear combination of terms with the form $\sin k\theta$.

   (b) Consider the annulus A defined as $a \leq r \leq b$ in polar coordinates, where $0 < a < b$. Show that for any integer k, the function $r^{\pm k} \sin k\theta$ is a solution to the Laplace equation $\nabla^2\phi = 0$ in A.

   Hint: the Laplacian in polar coordinates is given by

$$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}.$$

   (c) Find a solution to $\nabla^2\phi = 0$ in A that satisfies the boundary conditions

$$\phi(a, \theta) = 4\sin^3\theta, \qquad \phi(b, \theta) = 0.$$