

## Math 121A: Homework 8 (due April 10)

Note: there will be no regular office hours on Monday April 8th.

1. Consider the differential equation

$$y'' = f(x)$$

on the range  $-1 \leq x \leq 1$  subject to  $y(-1) = 0$  and  $y(1) = 0$ .

- (a) Calculate a Green function solution of the form

$$y(x) = \int_{-1}^1 G(x, x') f(x') dx'.$$

- (b) Plot  $G(x, x')$  on the range  $-1 \leq x \leq 1$  for the cases of  $x' = -2/3, -1/3, 0, 1/3, 2/3$ . Explain in words what the plotted functions represent.
- (c) Explicitly calculate the solution  $y(x)$  for the case when

$$f(x) = \begin{cases} 1 & \text{for } |x| < 1/4, \\ 0 & \text{for } |x| \geq 1/4. \end{cases}$$

Plot the solution, and explain how its form is related to the plots in part (b).

- (d) Explicitly calculate the solution for the case of  $f(x) = x$ , plot the solution, and check that the solution satisfies the differential equation and the boundary conditions.
2. Define

$$f(x) = \delta(x - 2) + \delta(x) + \delta(x + 2)$$

and

$$g(x) = \begin{cases} 1 - |x| & \text{for } |x| < 1, \\ 0 & \text{for } |x| \geq 1, \end{cases}$$

on  $-\infty < x < \infty$ . In addition, define  $h(x) = f(2x)$ . Calculate  $f * g$  and  $h * g$ , and plot them.

3. Consider the function

$$f_0(x) = \begin{cases} 1 & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Define  $f_{k+1} = f_k * f_0$ . Explicitly calculate the functions  $f_1, f_2$ , and  $f_3$  and plot them.

4. Consider the function

$$f(x) = \begin{cases} -a - x & \text{for } -a < x < 0, \\ a - x & \text{for } 0 \leq x < a, \\ 0 & \text{otherwise} \end{cases}$$

for  $-\pi < x < \pi$ , where  $0 \leq a < \pi$ . Plot the function and determine whether it is odd, even, or neither. Calculate the Fourier series of  $f$  and plot it for the cases of  $a = 1$  and  $a = 2$  using the first ten non-zero terms.

5. Boas exercises 14.2.22, 14.2.23, 14.2.24

6. Boas exercise 14.2.46

7. Boas exercise 14.2.47