Math 121A: Homework 4 (due February 20)

Office hours: Friday Feb 15, 2PM–3PM, Monday Feb 18, 4PM–6PM.

1. Boas exercise 3.5.31

2. Show that if λ is an eigenvalue of an orthogonal matrix A with eigenvector $\mathbf{v}$, then $\lambda = \pm 1$.

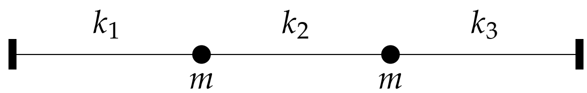
3. Boas exercise 3.6.6

4. Boas exercise 3.6.30

5. Find the eigenvalues and corresponding eigenvectors of the matrix

$$M = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

6. Consider the system shown below of two masses of mass m, coupled together between two fixed walls via springs with varying spring constants.



Let $x(t)$ and $y(t)$ be the horizontal displacements of the two masses as a function of time.

(a) Write down a system of differential equations for $\ddot{x}$ and $\ddot{y}$.

(b) For the case when $m = 1$, $k_1 = 1$, and $k_3 = 2$, calculate the eigenvalues associated with this system, which are associated with the characteristic frequencies of vibration.

(c) Plot the eigenvalues as a function of k_2 over the range $0 \leq k_2 \leq 3$. Discuss the physical interpretation of the changes to the eigenvalues as k_2 is increased.

7. (a) Let S be the set of solutions $y(t)$ to the differential equation $dy/dt = -y$ for $t \geq 0$. With addition and scalar multiplication of elements defined in the usual way, is S a vector space?

(b) Let T be the set of solutions $y(t)$ to the differential equation $dy/dt = 1 - y$ for $t \geq 0$. With addition and scalar multiplication of elements defined in the usual way, is T a vector space?

8. Boas exercise 3.9.19

9. Boas exercise 3.11.10