

Math 121A: Homework 1 (due February 1)

Office hours: Friday, Jan 24, 2PM–3PM; Thursday, Jan 31, 10AM–12PM. Note: for the exercises listed in the textbook, “Boas exercise $b.c.d$ ” refers to chapter b , section c , problem d .

1. Boas exercise 1.1.12
2. Boas exercise 1.2.4
3. Boas exercise 1.4.6
4. Two points P and Q are connected by a straight road of length d . At time $t = 0$, a car starts driving from P to Q at a constant speed c . At $t = 0$, a bee starts flying from Q toward the car at a constant speed $b > c$, and takes a zig-zagging path, reversing direction each time it meets the car or point Q . This process stops when the car reaches Q . Let a_1 be the distance the bee covers from Q to the car, a_2 be the distance it covers from the car back to Q , a_3 be the distance it covers from Q to meet the car the second time, and so on.
 - (a) Calculate the values of the infinite sequence a_1, a_2, a_3, \dots of distances that the bee covers.
 - (b) What is the total distance $D = \sum a_i$ covered by the bee? Why should this answer be expected?
 - (c) **Optional for the enthusiasts.** Suppose the bee travels at a different speed b' on the sections when it is moving from Q toward the car. What are the values of a_i and D in this case?
5. Assume that $\sum 1/n$ diverges. By using the comparison test with an appropriate series, determine whether $\sum_{n=1}^{\infty} 1/(n + \frac{1}{2})$ converges or diverges.
6. Boas exercise 1.6.8
7. Boas exercise 1.6.25
8. Boas exercise 1.7.5
9. Boas exercise 1.10.2
10. (a) Find the exact interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{n}.$$

- (b) Show that the power series

$$\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{y}{1+y^2} \right)^n$$

converges for all real numbers y .