Math 104: Homework 3 (due February 10)

1. Ross exercise 8.4

2. Ross exercise 8.5

3. Ross exercise 9.6

4. Let (a_n) be a sequence defined according to $a_1 = t$ where $t > 0$, and $a_{n+1} = 2a_n/(1 + a_n)$ for $n \in \mathbb{N}$. Prove that $a_n \to 1$ as $n \to \infty$.

5. Let (s_n) and (t_n) be Cauchy sequences defined on $\mathbb{R}$, and let (u_n) be a sequence defined as $u_n = as_n + bt_n$ for all n, where $a, b \in \mathbb{R}$. By using the definition of a Cauchy sequence only, without assuming that limits of (s_n) and (t_n) exist, prove that (u_n) is a Cauchy sequence.

6. Ross exercise 9.10

7. Ross exercise 10.8

8. Let (s_n) be a sequence defined for $n \in \mathbb{N}$ as

$$s_n = \begin{cases} 1 + \frac{1}{n} & \text{for } n \text{ odd,} \\ -1 & \text{for } n \text{ even.} \end{cases}$$

Calculate the monotonic sequences

$$u_N = \inf\{s_n : n > N\}, \qquad v_N = \sup\{s_n : n > N\}$$

for each $N \in \mathbb{N}$ and hence determine $\liminf s_n$ and $\limsup s_n$.