

Math 104: Homework 1 (due January 27)

1. Ross exercise 1.3
2. Ross exercise 1.8
3. The Fibonacci numbers are defined by $F_0 = 0$ and $F_1 = 1$, and

$$F_{n+1} = F_n + F_{n-1}$$

for $n \in \mathbb{N}$. Let the golden ratio be defined as $\varphi = \frac{1+\sqrt{5}}{2}$.

- (a) Show that $\varphi^2 = 1 + \varphi$.
- (b) Let

$$f(n) = \frac{\varphi^n - (1 - \varphi)^n}{\sqrt{5}}.$$

For $n \in \mathbb{N}$, define H_n to be the hypothesis that “both $F_n = f(n)$ and $F_{n-1} = f(n-1)$ ”. Apply mathematical induction to prove that H_n is true for all $n \in \mathbb{N}$, and deduce that $F_n = f(n)$ for all $n \in \mathbb{N} \cup \{0\}$. [Hint: it is simpler to carry out the algebra in terms of φ and use the identity in (a), as opposed to calculating explicitly in terms of $(1 + \sqrt{5})/2$.]

4. Ross exercise 2.4
5. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.
6. Ross exercise 3.1
7. Ross exercise 3.4
8. Ross exercise 3.5