Math 104: Midterm 2 information

- The second class midterm will take place on April 1st from 3:10pm–4pm, in Room 70 of Evans Hall.
- The exam will cover everything in class from metric spaces up to the lectures on power series before Spring Break. This corresponds to chapters 13, 17–20, and 23–27 in Ross.
- The exam is closed book – no textbooks, notebooks, or calculators allowed. As on the first midterm, you will not be expected to know every theorem by heart, but you will be expected to remember the definitions, such as metrics, open/closed sets, interiors and closures, uniform continuity, uniform convergence, and the radius of convergence for power series. You will also need to remember some of the key results, such as the Intermediate Value Theorem, and the theorem that a continuous function on a closed interval is bounded and achieves its bounds.
- There will not be any questions that specifically cover the material from the first midterm (*eg.* suprema, sequences, series). However, since many of the more recent topics build on this prior work, you will need to be familiar with this.
- There will be two short questions, followed by two longer questions involving mathematical proof.

Sample midterm questions

1. Let (f_n) be a sequence of uniformly continuous functions on an interval (a, b), and suppose that f_n converges uniformly to a function f. Prove that f is uniformly continous on (a, b).

2. Prove that the functions

$$d_1(x, y) = (x - y)^4, \qquad d_2(x, y) = 1 + |x - y|,$$

$$d_3(x, y) = \begin{cases} x - y & \text{if } x > y \\ 2(y - x) & \text{if } x \leq y \end{cases}$$

are not metrics on $\mathbb{R}$.

3. Find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{x^n}{n\sqrt{n}}, \qquad \sum_{n=0}^{\infty} 4^n \, x^{2n+1}, \qquad \sum_{n=0}^{\infty} x^{n^2}.$$

4. Suppose that f_n converges uniformly to f on a set $S \subseteq \mathbb{R}$, and that g is a bounded function on S. Prove that the multiplication $g \cdot f_n$ converges uniformly to $g \cdot f$.

5. Let (f_n) be a sequence of bounded functions on a set S, and suppose that $f_n \to f$ uniformly on S. Prove that f is a bounded function on S.

6. Let (f_n) be a sequence of real-valued continuous functions defined on the interval $[0, 1]$. Suppose that f_n converges uniformly to a function f. Define a global bound M according to

$$M = \sup\{|f_n(x)| \ : \ n \in \mathbb{N}, \ x \in [0, 1]\}.$$

Prove that M is finite.