Math 104: Midterm information

- The first class midterm will take place on February 25th from 3:10pm–4pm, in Room 156 of Dwinelle Hall.
- The exam will cover everything in class up to and including alternating series. This corresponds to chapters 1–5, 7–12, 14, and 15 in Ross. The more recent material on metric spaces will not be covered.
- The exam is closed book – no textbooks, notebooks, or calculators allowed. You will not be expected to know every theorem by heart, but you will be expected to remember the basic definitions, such as suprema/infima, convergence/divergence, lim sup, lim inf, and subsequential limits. You should also be familiar with the limit theorems for determining convergence of sequences, and the tests for determining series convergence and divergence.
- There will be two short questions, followed by two longer questions which will involve constructing a mathematical proof.
- If a question uses the word "prove", then you will be expected to write down a mathematical proof similar to those in the textbook or given in the homework solutions. If a question uses weaker langauge, such as "determine", "justify", or "compute", a less rigorous argument will still receive full credit.
- You can assume a number of basic results, such as
  - $\sum n^{-p}$ diverges if and only if $p > 1$,
  - the triangle inequality: $|a| + |b| \geq |a + b|$ for all $a, b \in \mathbb{R}$,
  - $\lim_{n\to\infty} a^n = 0$ for $|a| < 1$,
  - $|b| < a$ if and only if $-a < b < a$.

Sample midterm questions

The following questions are of a similar style to the ones that will be on the midterm. They are designed to test familiarity with basic concepts, and will generally be more straightforward than some of the questions on the homework.

1. Prove that $1 + \sqrt{1 + \sqrt{2}}$ is irrational.
2. Consider the following series defined for $n \in \mathbb{N}$:

$$\sum \frac{8^n}{(n!)^2}, \qquad \sum \frac{(-1)^n}{\sqrt{n^2 + n}}.$$

For each series, determine whether they converge or diverge. If you make use of any of the theorems for determining series properties, you should state which one you use.

3. (a) Let S and T be non-empty bounded subsets of $\mathbb{R}$. Prove that $\sup S \cup T = \max\{\sup S, \sup T\}$ and $\sup S \cap T \leq \min\{\sup S, \sup T\}$.
   (b) Extend part (a) to the cases where S and T are not bounded.
   (c) Give an example where $\sup S \cap T < \min\{\sup S, \sup T\}$.

4. Suppose that (s_n) is a convergent sequence and (t_n) is a sequence that diverges to ∞. Prove that
$$\lim_{n\to\infty} s_n + t_n = \infty.$$

5. (a) Let (s_n) and (t_n) be two sequences defined for $n \in \mathbb{N}$. Prove that $\limsup s_n + \limsup t_n \geq \limsup s_n + t_n$.
   (b) Construct an example where $(\limsup s_n) \cdot (\limsup t_n) \neq \limsup(s_n t_n)$.

6. Let (a_n) and (b_n) are sequences defined for $n \in \mathbb{N}$. Suppose that $a_n \to a$ and $b_n \to b$ as $n \to \infty$ for some $a, b \in \mathbb{R}$. If $a_n \leq b_n$ for all n, show that $a \leq b$.