Math 104: Homework 7 (due March 18)

1. Ross exercise 19.6
2. (a) Let S be a subset of $\mathbb{R}$, and let $f : S \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be uniformly continuous functions. Prove that the composition $g \circ f : S \to \mathbb{R}$ is uniformly continuous.
   (b) Let f and g be two uniformly continuous functions from S to $\mathbb{R}$. Prove that $f + g$ is uniformly continuous.
   (c) Show that there exist uniformly continuous functions f and g from S to $\mathbb{R}$ such that the multiplication $f \cdot g$ is not uniformly continuous.
3. Ross exercise 20.12
4. Ross exercise 20.16
5. Ross exercise 23.5
6. Ross exercise 24.2
7. Ross exercise 24.16
8. Let f be a real-valued continuous function. Recall that for any subset $S \subseteq \mathbb{R}$, then $f(S)$ is defined as $\{f(x) : x \in S\}$. Suppose that $f(I)$ is open for any open interval I. Prove that f is monotonic.