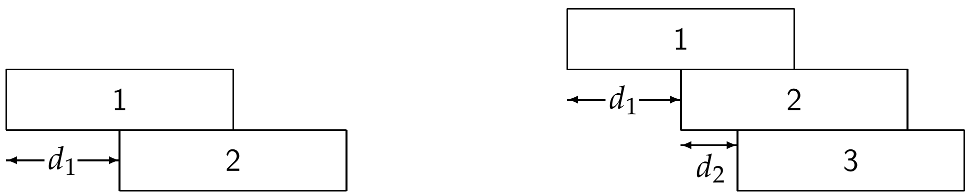
Math 104: Homework 4 (due February 18)

1. Ross exercise 11.10
2. Ross exercise 12.10
3. Ross exercise 14.13
4. Determine the convergence or divergence of each of the following series defined for $n \in \mathbb{N}$:

$$\text{(a)} \sum_n \frac{n^3}{2^n}, \quad \text{(b)} \sum_n \sqrt{n+1} - \sqrt{n}, \quad \text{(c)} \sum_n \frac{1}{\sqrt{n!}}, \quad \text{(d)} \sum_n 2^{-3n+(-1)^n}, \quad \text{(e)} \sum_n \frac{n!}{n^n}.$$

5. Let (a_n) be a sequence where $0 \le a_n < 1$ for $n \in \mathbb{N}$. Prove that if $\sum_{n=1}^{\infty} a_n$ converges, then so do $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} a_n/(1-a_n)$. Are the converse statements true?
6. Let (u_n) and (v_n) be sequences of positive real numbers for $n \in \mathbb{N}$. For each of the following statements, either prove it or provide a counterexample.
   (a) If (u_n) and (v_n) are equal apart from at finitely many n, then $\sum u_n$ or $\sum v_n$ either both converge or both diverge.
   (b) If (u_n) and (v_n) are equal at infinitely many n, then $\sum u_n$ or $\sum v_n$ either both converge or both diverge.
   (c) If $(u_n/v_n) \to 1$ as $n \to \infty$, then $\sum u_n$ and $\sum v_n$ both converge or both diverge.
   (d) If $u_n - v_n \to 0$, then $\sum u_n$ and $\sum v_n$ both converge or both diverge.
   (e) If $(u_{n+1}/u_n) > k > 1$ for infinitely many n, then $\sum u_n$ diverges.
7. Find a sequence (a_n) such that $\sum_{n=1}^{2N} a_n$ and $\sum_{n=1}^{2N+1} a_n$ both converge as $N \to \infty$, but $\sum a_n$ is divergent.
8. **Optional for the enthusiasts.** Consider an infinite number of bricks of unit length, made from a uniform material.



(a) (b)

Begin by considering diagram (a): what is the maximum distance d_1 that brick 1 can overhang brick 2 without falling? Now, by considering combined center of mass of bricks 1 and 2, find the distance d_2 that can they can overhang brick 3. Now determine the maximum distance d_n that a stack of bricks from 1 to n can overhang a brick (n+1). Does $\sum d_n$ converge or diverge?