Math 104: Homework 3 (due February 11)

1. Ross exercise 8.10

2. Ross exercise 9.6

3. Let (a_n) be a sequence defined according to $a_1 = t$ where $t > 0$, and $a_{n+1} = 2a_n/(1 + a_n)$ for $n \in \mathbb{N}$. Prove that $a_n \to 1$ as $n \to \infty$.

4. Let (s_n) and (t_n) be Cauchy sequences defined on $\mathbb{R}$, and let (u_n) be a sequence defined as $u_n = as_n + bt_n$ for all n, where $a, b \in \mathbb{R}$. By using the definition of a Cauchy sequence only, without assuming that limits of (s_n) and (t_n) exist, prove that (u_n) is a Cauchy sequence.

5. Ross exercise 9.10

6. Ross exercise 10.6

7. Ross exercise 10.8

8. Let (s_n) be a sequence defined by $s_n = (-1)^n(1 + \frac{1}{n})$. Prove that $\limsup s_n = 1$ and $\liminf s_n = -1$.